SOLUTIONS TO EXERCISES

[Last updated 7/7/2015]

Chapter 4.

1. Determine the dual of the given minimization problem.

$$\begin{aligned} \textit{max} \quad & w = 4y_1 + 4y_2 \\ \textit{subject to} \quad & 2y_1 + y_2 \leq 3 \\ & y_1 + 2y_2 \leq 3 \\ & y_1,\ y_2 \geq 0 \end{aligned}$$

2. Determine the dual of the given minimization problem.

$$\begin{aligned} \textit{max} \quad & w = 23y_1 + 10y_2 + 40y_3 \\ \textit{subject to} \quad & 3x_1 + 1x_2 + 8x_3 \leq 4 \\ & 2x_1 + x_3 \leq 1 \\ & \mathrm{x}_1 + x_2 + 2x_3 \leq 1 \\ & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

3. Solve the given problem using dual simplex method and use simplex method to verify optimality.

   Step 1: Adding slack variable to form a dual feasible solution

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	5	35	20	0	0	0
x_4	0	1	−1	−1	1	0	−2
x_5	0	−1	−3	0	0	1	−3

   Step 2:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	0	20	20	0	5	−15
x_4	0	0	−4	−1	1	1	−5
x_1	0	1	3	0	0	−1	3

   Step 3:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	0	0	15	5	10	−40
x_2	0	0	1	0.25	−0.25	−0.25	1.25

x_1	0	1	0	-0.75	0.75	-0.25	-0.75

Step 4:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	20	0	0	20	5	-55
x_2	0	0.33	1	0	0	-0.33	1
x_3	0	-1.33	0	1	-1	0.33	1

4. Solve the given problem using dual simplex method and use simplex method to verify optimality.

   Step 1: Adding slack variable to form a dual feasible solution

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	2	1	0	0	0	0
x_4	0	-2	1	1	1	0	-4
x_5	0	1	2	-1	0	1	-6

   Step 2:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	2	1	0	0	0	0
x_4	0	-1	3	0	1	1	-10
x_3	0	-1	-2	1	0	-1	6

   Step 3:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	0	-5	0	-2	-2	-20
x_1	0	1	-3	0	-1	-1	10
x_3	0	0	-5	1	-1	0	16

5. Solve the given problem using dual simplex method and use simplex method to verify optimality.

   Step 1: Adding slack variable to form a dual feasible solution

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	4	12	18	0	0	0
x_4	0	-1	0	-3	1	0	-3
x_5	0	0	-2	-2	0	1	-5

   Step 2:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	4	0	6	0	6	-30
x_4	0	-1	0	-3	1	0	-3
x_2	0	0	1	1	0	-0.5	2.5

Step 3:

	x_0	x_1	x_2	x_3	x_4	x_5	RHS
x_0	1	2	0	0	2	6	-36
x_3	0	0.33	0	1	-0.33	0	1
x_2	0	-0.33	1	0	0.33	-0.5	1.5

6. Solve the given problem using dual simplex method and use simplex method to verify optimality.

   Step 1: Adding slack variable to form a dual feasible solution

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_0	1	-2	-1	0	0	0	0	0
x_3	0	-1	0	1	0	0	0	-2
x_4	0	3	4	0	1	0	0	24
x_5	0	-4	-3	0	0	1	0	-12
x_6	0	1	-2	0	0	0	1	-1

   Step 2:

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_0	1	$-2/3$	0	0	0	0	$-1/3$	4
x_3	0	-1	0	1	0	0	0	-2
x_4	0	$-7/3$	0	0	1	$4/3$	0	8
x_2	0	$4/3$	1	0	0	$-1/3$	0	4
x_6	0	$11/3$	0	0	0	$-2/3$	1	7

   Step 3:

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_0	1	0	0	$-2/3$	0	$-1/3$	0	$16/3$
x_1	0	1	0	-1	0	0	0	2
x_4	0	0	0	$-7/3$	1	$4/3$	0	$38/3$
x_2	0	0	1	$4/3$	0	$-1/3$	0	$4/3$
x_6	0	0	0	$11/3$	0	$-2/3$	1	$-1/3$

   Step 4:

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_0	1	0	0	$5/2$	0	0	$-1/2$	$11/2$
x_1	0	1	0	-1	0	0	0	2
x_4	0	0	0	5	1	0	2	12
x_2	0	0	1	$-1/2$	0	0	$-1/2$	$3/2$
x_5	0	0	0	$-11/2$	0	1	$-3/2$	$1/2$